**13.4 Expected Value and Fairness**

The **expected value** is an average expectation per game if the game is played many times!

\*The expected return per game

\*Payoff= amount earned

A game gives payoffs of a1, a2, a3…an with the probabilities p1, p2, p3…pn

The expected value (orexpectation) E of this game is:

$$E=a\_{1}p\_{1}+a\_{2}p\_{2}+…+a\_{n}p\_{n}$$

**NOTE**! Expected value can be negative! A negative expected value indicates a negative payout (i.e. you’re losing money!)

**Fairness**: Occurs when the probability of winning is equally as likely (meaning you have the same chance of winning and losing) or when expected value is such that a player can “break even” (meaning that after playing a game numerous times, his returns will match what he pays to play the game, E=0)

Ex 1: A coin is flipped. Heads, you win $1. Tails, you lose $1. What is the expected value of this game?

*A game whose expected winnings are \_\_\_\_\_\_\_ is called a \_\_\_\_\_\_\_\_\_\_ \_\_\_\_\_\_\_\_\_\_.*

Ex 2: Jane gets $6 if a die shows a 6 and loses $1 otherwise. What is her expectation?

Ex 3: A die is rolled. If the die shows a 1, 2, or 3 you get 10 points. If the die shows a 4 or a 5, you lose 13 points. If the die shows a 6, you lose 1 point. What is the expected value of this game?

Ex 4: In Monte Carlo, the game of roulette is played on a wheel with slots numbered 0, 1, 2, …, 36. The wheel spun and a ball dropped on the wheel is equally likely to end up in any one of the slots. To play the game, you bet $1 on any number. If the ball stops in your slot, you win $36 (the $1 you bet plus $35). Find the expected value of this game.

Ex 5: A sweepstakes contest offers a first prize of one million dollars, a second prize of $200,000, and a third prize of $40,000. Suppose that three million people enter the contest and three names are selected randomly for the three prizes.

1. What are the expected winnings of a person participating in this contest?
2. Is it worth paying $0.50 to enter this contest?

Ex 6: Real Life Ex: A life insurance policy for a 40-year old woman will pay $10,000 if she dies within 1 year. The policy costs $300. Statistics (namely, mortality tables) indicate that the relative frequency of a 40-year old woman dying within 1 year is 0.02. What is the expected profit of this policy to the woman?

Ex. 7: It costs a company $5 to make a layer cake that sells for $20 the first day they are baked. If a cake is not sold the first day, the new price is $15. The probability of selling the cake the first day is 60%. There is a 20% probability of selling the cake on the second day. If the cake doesn’t sell by the end of the second day, it is donated. What is the approximate expected profit for the bakery on the sale of its layer cakes?